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An unorthodox alternative for righthanded neutrinos: lefthanded see–saw*

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Abstract

A new lefthanded see–saw mechanism is constructed, implying both the smallness of active–neutrino masses and decoupling of heavy passive neutrinos, similarly to the situation in the case of conventional see–saw. But now, in place of the conventional righthanded neutrinos, the lefthanded sterile neutrinos play the role of heavy passive neutrinos, the righthanded neutrinos and righthanded sterile neutrinos being absent. Here, the lefthanded sterile neutrinos are different from charge conjugates of conventional righthanded neutrinos because their lepton numbers differ. In this case, the neutrino mass term is necessarily of pure Majorana type.

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As is well known, the popular see–saw mechanism [1] provides us with a gentle way of introducing small neutrino masses into the original Standard Model , where Dirac–type masses are zero due to the absence of righthanded neutrinos. In fact, the actual righthanded neutrinos (of all three generations) get in this mechanism large (by assumption) Majorana–type masses and so, become practically decoupled from lefthanded neutrinos that are allowed to carry only small Majorana–type masses. Such large righthanded Majorana masses are, on the other hand, related to an expected high energy scale at which lepton number violation ought to appear.

In this note, we present a new, *a priori* possible mechanism, where the role played in the see–saw mechanism by righthanded neutrinos is taken over by hypothetic *lefthanded* sterile neutrinos free of any Standard Model charges. Thus, the new mechanism may be called *lefthanded see–saw*. In this case, both righthanded neutrinos and righthanded sterile neutrinos are conjectured to be absent, $\nu_R \equiv 0$ and $\nu_R^{(s)} \equiv 0$, so that

$$\nu \equiv \nu_L \;\; , \;\; \nu^{(s)} \equiv \nu_L^{(s)} \tag{1}$$

are active and sterile neutrinos, respectively (for all three neutrino generations).

At first sight it may seem that it is sufficient to reinterpret the sterile neutrino $\nu_L^{(s)}$ as the charge conjugate $(\nu_R)^c$ of conventional righthanded neutrino ν_R in order to return to the safe ground of familiar see–saw. But such a reinterpretation is impossible if $\nu_L^{(s)}$, like ν_L, has the lepton number $L = +1$ (this would be also excluded if its L were 0). In this argument L is assumed to be well defined, even when it is violated.

In the new situation, the neutrino mass term is of pure Majorana type

$$\begin{aligned} -\mathcal{L}_{\rm mass} &= \frac{1}{2}\left(\overline{(\nu_L)^c}\,,\,\overline{(\nu_L^{(s)})^c}\right)\left(\begin{array}{cc} m^{(L)} & \mu^{(L)} \\ \mu^{(L)} & m_s^{(L)} \end{array}\right)\left(\begin{array}{c} \nu^L \\ \nu_L^{(s)} \end{array}\right) + \text{h.c.} \\ &= \frac{1}{2}\left(\overline{(\nu_M)^c}\,,\,\overline{(\nu_M^{(s)})^c}\right)\left(\begin{array}{cc} m^{(L)} & \mu^{(L)} \\ \mu^{(L)} & m_s^{(L)} \end{array}\right)\left(\begin{array}{c} \nu^M \\ \nu_M^{(s)} \end{array}\right) \;, \end{aligned} \tag{2}$$

where

$$\nu_M \equiv \nu_L + (\nu_L)^c \;\; , \;\; \nu_M^{(s)} \equiv \nu_L^{(s)} + (\nu_L^{(s)})^c \tag{3}$$

(in the case of one neutrino generation). In the case of three neutrino generations, real numbers $m^{(L)}$, $m_s^{(L)}$ and $\mu^{(L)}$ become 3×3 matrices $\widehat{m}_s^{(L)}$, $\widehat{m}_s^{(L)}$ and $\widehat{\mu}^{(L)}$, real and symmetric for simplicity, while neutrino fields ν_L and $\nu_L^{(s)}$ transit into the field columns $\vec{\nu}_L = (\nu_{e\,L}\,,\,\nu_{\mu\,L}\,,\,\nu_{\tau\,L})^T$ and $\vec{\nu}_L^{(s)} = \left(\nu_{e\,L}^{(s)}\,,\,\nu_{\mu\,L}^{(s)}\,,\,\nu_{\tau\,L}^{(s)}\right)^T$.

In Eq. (2), all four terms violate lepton number ($\Delta L = \pm 2$), the terms proportional to $\mu^{(L)}$ and $m^{(L)}$ break electroweak symmetry, while the term proportional to $m_s^{(L)}$ is electroweak gauge invariant. The terms $\mu^{(L)}$ and $m^{(L)}$ may be generated spontaneously by Higgs mechanism, the first — by the linear Higgs coupling (of the Majorana type):

$$\mathcal{L}_H = \frac{1}{2} g_H \left[\overline{(l_L)^c} H \nu_L^{(s)} - \overline{l_L} H^c (\nu_L^{(s)})^c\right] + \text{h.c.}\;\;,\;\; \mu^{(L)} \equiv g_H \langle H^\circ \rangle \tag{4}$$

and the second — by the familiar bilinear effective Higgs coupling (also of the Majorana type):

$$\mathcal{L}_{HH} = \frac{1}{4M} g_{HH} \left[\overline{(l_L)^c}\vec{\tau} l_L\right] \cdot \left(H^T i\tau_2 \vec{\tau} H\right) + \text{h.c.}\;\;,\;\; m^{(L)} \equiv \frac{1}{M} g_{HH} \langle H^\circ \rangle^2 \tag{5}$$

(see *e.g.* Ref. [2]). Here, $\overline{(l_L)^c} = (l_L)^T C^{-1} i\tau_2$ and

$$l_L = \begin{pmatrix} \nu_L \\ l_L^- \end{pmatrix} \;\;,\;\; H = \begin{pmatrix} H^+ \\ H^\circ \end{pmatrix} \;,$$
$$(l_L)^c = i\tau_2 \begin{pmatrix} (\nu_L)^c \\ (l_L^-)^c \end{pmatrix} = \begin{pmatrix} (l_L^-)^c \\ -(\nu_L)^c \end{pmatrix} \;\;,\;\; H^c = i\tau_2 \begin{pmatrix} H^{+\,c} \\ H^{\circ\,c} \end{pmatrix} = \begin{pmatrix} H^{\circ\,c} \\ -H^{+\,c} \end{pmatrix} \;, \tag{6}$$

with $(\nu_L)^c = C\overline{\nu}_L^T$, $(l_L^-)^c = C\overline{l_L^-}^T$ and $H^{+\,c} = H^{+\,\dagger} = H^-$, $H^{\circ\,c} = H^{\circ\,\dagger}$ $(H^{c\,\dagger} = -H^T i\tau_2)$. In Eq. (5), M is a large mass scale probably related to the GUT scale, so that the inequality $m^{(L)} \ll \mu^{(L)}$ is plausible. Note that the lefthanded sterile neutrino $\nu_L^{(s)}$, a Standard Model scalar, may be an SU(5) scalar. However, it cannot be an SO(10) covariant (say, a scalar or a member of 16–plet), since the SO(10) formula $Y = 2I_3^{(R)} + B - L$ for weak hypercharge does not work in the case of $\nu_L^{(s)}$ with $L = 1$ and $B = 0$ ($Y = 0$ and $I_3^{(R)} = 0$ imply $B - L = 0$). Thus, the existence of $\nu_L^{(s)}$ breaks dynamically the SO(10) symmetry, unless $\nu_L^{(s)}$ gets $B - L = 0$ (*e.g.* $L = 0 = B$ or $L = 1 = B$), when it may be an SO(10) scalar (but, in absence of ν_R, the 16–plet is still not completed). If $L = 0 = B$, then of the intrinsic quantum numbers only spin 1/2 and chirality –1 are carried by $\nu_L^{(s)}$.

After its diagonalization, the mass term (2) becomes

$$- \mathcal{L}_{\rm mass} = \frac{1}{2} \left(\overline{\nu}_I \, , \, \overline{\nu}_{II} \right) \left(\begin{array}{cc} m_I & 0 \\ 0 & m_{II} \end{array} \right) \left(\begin{array}{c} \nu_I \\ \nu_{II} \end{array} \right) \; , \tag{7}$$

where

$$\begin{aligned} \nu_I &= \nu_M \cos\theta - \nu^{(s)}_M \sin\theta \; , \\ \nu_{II} &= \nu_M \sin\theta + \nu^{(s)}_M \cos\theta \end{aligned} \tag{8}$$

with $\tan\theta = (m_{II} - m^{(L)}_s)/\mu^{(L)}$, and

$$m_{I,\,II} = \frac{m^{(L)} + m^{(L)}_s}{2} \mp \sqrt{\left(\frac{m^{(L)} - m^{(L)}_s}{2} \right)^2 + \mu^{(L)\,2}} \; . \tag{9}$$

Assuming that

$$0 \leq m^{(L)} \ll \mu^{(L)} \ll m^{(L)}_s \; , \tag{10}$$

we obtain the neutrino mass eigenstates

$$\nu_I \simeq \nu_M - \frac{\mu^{(L)}}{m^{(L)}_s} \nu^{(s)}_M \simeq \nu_M \; , \; \nu_{II} \simeq \nu^{(s)}_M + \frac{\mu^{(L)}}{m^{(L)}_s} \nu_M \simeq \nu^{(s)}_M \tag{11}$$

related to the neutrino masses

$$m_I \simeq - \frac{\mu^{(L)\,2}}{m^{(L)}_s} \; , \; m_{II} \simeq m^{(L)}_s \; . \tag{12}$$

Thus, $|m_I| \ll m_{II}$, what practically decouples $\nu^{(s)}_L$ from ν_L. Here, the minus sign at m_I is evidently irrevelant for ν_I which, as a relativistic particle, is kinematically characterized by m^2_I.

New Eqs. (10) — (12) give us a purely lefthanded counterpart of the popular see–saw mechanism, where the assumption

$$0 \leq m^{(L)} \ll m^{(D)} \ll m^{(R)} \tag{13}$$

implies the neutrino mass eigenstates

$$\nu_I \simeq \nu_M \equiv \nu_L + (\nu_L)^c \; , \; \nu_{II} \simeq \nu'_M \equiv \nu_R + (\nu_R)^c \tag{14}$$

connected with the neutrino masses

$$m_I \simeq -\frac{m^{(D)\,2}}{m^{(R)}}\;,\; m_{II} \simeq m^{(R)}\;. \tag{15}$$

In that case, the neutrino mass term has the form

$$\begin{aligned} -\mathcal{L}^{\rm conv}_{\rm mass} &= \frac{1}{2}\left(\overline{(\nu_L)^c}\,,\,\overline{\nu_R}\right)\begin{pmatrix} m^{(L)} & m^{(D)} \\ m^{(D)} & m^{(R)}\end{pmatrix}\begin{pmatrix}\nu_L \\ (\nu_R)^c\end{pmatrix} + \text{h.c.} \\ &= \frac{1}{2}\left(\overline{\nu_M}\,,\,\overline{\nu'_M}\right)\begin{pmatrix} m^{(L)} & m^{(D)} \\ m^{(D)} & m^{(R)}\end{pmatrix}\begin{pmatrix}\nu_M \\ \nu'_M\end{pmatrix}\;. \end{aligned} \tag{16}$$

So, from Eq. (15) $|m_I| \ll m_{II}$, what practically leads to decoupling ν_R from ν_L. But, while in Eq. (15) the magnitude of neutrino Dirac mass $m^{(D)}$ may be compared with the mass of corresponding charged lepton, in Eq. (12) the magnitude of $\mu^{(L)}$, responsible for the coupling $(1/2)\mu^{(L)}\left[\overline{(\nu_L)^c}\nu^{(s)}_L + \overline{\nu_L}(\nu^{(s)}_L)^c\right]$ + h.c., may be quite different (perhaps smaller).

In the general case of three neutrino generations, Eqs. (11) and (12) are replaced by

$$\vec{\nu}_I \simeq \vec{\nu}_M \equiv \vec{\nu}_L + (\vec{\nu}_L)^c\;,\; \vec{\nu}_{II} \simeq \vec{\nu}^{(s)}_M \equiv \vec{\nu}^{(s)}_L + (\vec{\nu}^{(s)}_L)^c \tag{17}$$

and

$$\widehat{m}_I \simeq -\widehat{\mu}^{(L)}\left(\widehat{m}^{(L)}_s\right)^{-1}\widehat{\mu}^{(L)}\;,\; \widehat{m}_{II} \simeq \widehat{m}^{(L)}_s \tag{18}$$

(compare *e.g.* Ref. [3]). Here, $\vec{\nu}_L = (\nu_{\alpha\,L})$, $\vec{\nu}_I = (\nu_{I\,\alpha})$, $\widehat{m}_I = (m_{I\,\alpha\beta})$, *etc.* $(\alpha\,,\,\beta = e\,,\,\mu\,,\,\tau)$. The Hermitian mass matrices $\widehat{m}_I$ and $\widehat{m}_{II}$ ($\widehat{m}^{(L)}$, $\widehat{m}^{(L)}_s$ and $\widehat{\mu}^{(L)}$ were taken real and symmetric for simplicity) can be diagonalized with the use of unitary matrices $\widehat{U}_I = (U_{I\,\alpha\,i})$ and $\widehat{U}_{II} = (U_{II\,\alpha\,i})$, respectively, giving the neutrino mass eigenstates

$$\begin{aligned} \nu_{I\,i} &= \sum_\alpha \left(\widehat{U}^\dagger_I\right)_{i\,\alpha}\nu_{I\,\alpha} = \sum_\alpha U^*_{I\,\alpha\,i}\nu_{I\,\alpha}\;, \\ \nu_{II\,i} &= \sum_\alpha \left(\widehat{U}^\dagger_{II}\right)_{i\,\alpha}\nu_{II\,\alpha} = \sum_\alpha U^*_{II\,\alpha\,i}\nu_{II\,\alpha} \end{aligned} \tag{19}$$

and the corresponding neutrino masses

$$\begin{aligned}
\delta_{ij} m_{I\,i} &= \sum_{\alpha\beta} \left(\widehat{U}_I^\dagger\right)_{i\,\alpha} m_{I\,\alpha\beta} \left(\widehat{U}_I\right)_{\beta\,j} = \sum_{\alpha\beta} U^*_{I\,\alpha\,i} U_{I\,\beta\,j} m_{I\,\alpha\beta}\;, \\
\delta_{ij} m_{II\,i} &= \sum_{\alpha\beta} \left(\widehat{U}_{II}^\dagger\right)_{i\,\alpha} m_{II\,\alpha\beta} \left(\widehat{U}_{II}\right)_{\beta\,j} = \sum_{\alpha\beta} U^*_{II\,\alpha\,i} U_{II\,\beta\,j} m_{II\,\alpha\beta}
\end{aligned} \tag{20}$$

$(i,j=1,2,3)$. In our case, $U_{I\alpha i}$ and $U_{II\alpha i}$ are real, while $m_{I\,\alpha\beta} \simeq -\sum_{\gamma\delta} \mu^{(L)}_{\alpha\gamma}\, (\widehat{m}_s^{(L)\,-1})_{\gamma\delta} \mu^{(L)}_{\delta\beta}$ and $m_{II\,\alpha\beta} \simeq m^{(L)}_{s\alpha\beta}$ due to Eqs. (18). When deriving Eq. (19), we assume that in the original lepton Lagrangian the charged–lepton mass matrix is diagonal and so, its diagonalizing unitary matrix $\widehat{U}^{(l)}$ is trivially equal to the unit matrix. Then, the lepton counterpart of the Cabibbo–Kobayashi–Maskawa matrix $\widehat{V} = \widehat{U}^{(\nu)\,\dagger}\widehat{U}^{(l)}$ becomes equal to $\widehat{U}^{(\nu)\,\dagger} = \widehat{U}_I^\dagger$, thus $V_{i\alpha} = U^*_{I\alpha i}$.

From Eqs. (17) and (19) we conclude for neutrino fields that

$$\begin{aligned}
\nu_{\alpha\,L} &\simeq \left(\nu_{I\,\alpha}\right)_L = \sum_i U_{I\,\alpha\,i} \left(\nu_{I\,i}\right)_L\;, \\
\nu^{(s)}_{\alpha\,L} &\simeq \left(\nu_{II\,\alpha}\right)_L = \sum_i U_{II\,\alpha\,i} \left(\nu_{II\,i}\right)_L\;.
\end{aligned} \tag{21}$$

Thus, the oscillation probabilities (on the energy shell) for active–neutrino states read (in the vacuum):

$$P(\nu_\alpha \to \nu_\beta) = |\langle \nu_\beta | e^{iPL} | \nu_\alpha \rangle|^2 \simeq \delta_{\alpha\beta} - 4 \sum_{i<j} U^*_{I\beta j} U_{I\alpha j} U_{I\beta i} U^*_{I\alpha i} \sin^2\left(1.27 \frac{m^2_{I\,j} - m^2_{I\,i}}{E} L\right)\;, \tag{22}$$

where $E = \sqrt{p_i^2 + m^2_{I\,i}}$ $(i = 1,2,3)$.

If the matrix $\widehat{m}_s^{(L)}$ happens to be nearly diagonal and has nearly degenerate eigenvalues: $m^{(L)}_{s\alpha\beta} \simeq \delta_{\alpha\beta} m^{(L)}_s$, then

$$m_{I\alpha\beta} \simeq -\sum_\gamma \frac{\mu^{(L)}_{\alpha\gamma}\mu^{(L)}_{\gamma\beta}}{m^{(L)}_s}\;,\; m_{II\alpha\beta} \simeq \delta_{\alpha\beta} m^{(L)}_s \tag{23}$$

and Eqs. (20) give

$$m_{Ii} \simeq -\frac{\mu_i^{(L)\,2}}{m_s^{(L)}}\;,\; m_{IIi} \simeq m_s^{(L)}\;, \tag{24}$$

where the eigenvalues $\mu_i^{(L)}$ of $\widehat{\mu}^{(L)} = (\mu_{\alpha\beta}^{(L)})$ are produced with the use of $\widehat{U}_I$:

$$\delta_{ij}\mu_i^{(L)} = \sum_{\alpha\beta} U^*_{I\,\alpha\,i} U_{I\,\beta\,j} \mu_{\alpha\beta}^{(L)}\ . \tag{25}$$

In this case, $\nu_{II\alpha}$ $(\alpha = e\,,\,\mu\,,\,\tau)$ are (approximate) neutrino heavy mass eigenstates decoupled from ν_{Ii} $(i = 1, 2, 3)$.

Concluding, the lefthanded see–saw, constructed in this note, implies both the smallness of active–neutrino masses and decoupling of heavy passive neutrinos, similarly to the situation in the case of conventional see–saw. However, introducing as heavy passive neutrinos the lefthanded sterile neutrinos $\nu_{\alpha L}^{(s)}$ in place of the conventional righthanded neutrinos $\nu_{\alpha R}$ $(\alpha = e\,,\,\mu\,,\,\tau)$, the new see–saw mechanism, when spoiling the chiral left–right pattern of original Standard Model for neutrinos, does it in a way different from the conventional see–saw mechanism. Recall that in the lefthanded see–saw the righthanded neutrinos continue to be completely absent.

Note finally that in the case of lefthanded see–saw the active and sterile neutrinos, as practically unmixed, cannot oscillate into each other. They could, if in Eq. (2) in place of the inequality (10) the relation

$$\mid m^{(L)} - m_s^{(L)} \mid \ll \mu^{(L)} \tag{26}$$

were conjectured, leading to their nearly maximal mixing. This mechanism, however, being a formal analogy of the conventional pseudo–Dirac case [4], would not imply automatically small neutrino masses, since then $m_{I,II} \simeq (1/2)(m^{(L)} + m_s^{(L)}) \mp \mu^{(L)}$ from Eq. (9). The smallness of m_I would require $(1/2)(m^{(L)} + m_s^{(L)}) \simeq \mu^{(L)}$.

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References

1. M. Gell–Mann, P. Ramond and R. Slansky, in *Supergravity* , ed. P. van Nieuwenhuizen and D.Z. Freedman, North–Holland, Amsterdam, 1979; T. Yanagida, in *Proc. of the Workshop on the Unified Theory of the Baryon Number in the Universe*, ed. O. Sawada and A. Sugamoto, KEK report No. 79–18, Tsukuba, Japan, 1979; see also R. Mohapatra and G. Senjanovic, *Phys. Rev. Lett.* **44**, 912 (1980).

2. R.D. Peccei, hep–ph/9906509.

3. G. Altarelli and F.Feruglio, CERN–TH/99–129 + DFPD–99/TH/21, hep–ph/9905536.

4. See *e.g.* W. Królikowski, hep–ph/9910308.